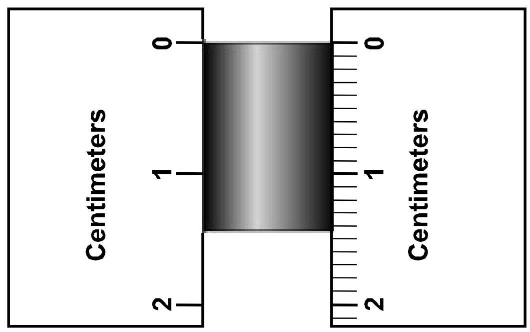
Numbers in Science
Exploring Measurements, Significant Digits, and Dimensional Analysis

TAKING MEASUREMENTS

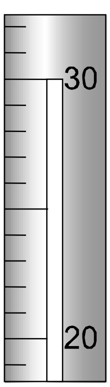
The accuracy of a measurement depends on two factors: the skill of the individual taking the measurement and the capacity of the measuring instrument. When making measurements, you should always read to the smallest mark on the instrument and then estimate another digit beyond that.



For example, if you are reading the length of the steel pellet pictured above using only the ruler shown to the left of the pellet, you can confidently say that the measurement is between 1 and 2 centimeters. However, you MUST also include one additional digit estimating the distance between the 1 and 2 centimeter marks. The correct measurement for this ruler should be reported as 1.5 centimeters. It would be incorrect to report this measurement as 1 centimeter or even 1.50 centimeters given the scale of this ruler.

What if you are using the ruler shown on the right of the pellet? What is the correct measurement of the steel pellet using this ruler? 1.4 centimeters? 1.5 centimeters? 1.40 centimeters? 1.45 centimeters? The correct answer would be 1.45 centimeters. Since the smallest markings on this ruler are in the tenths place we must carry our measurement out to the hundredths place.

If the measured value falls exactly on a scale marking, the estimated digit should be zero.



The temperature on this thermometer should read 30.0°C. A value of 30°C would imply this measurement had been taken on a thermometer with markings that were 10° apart, not 1° apart.

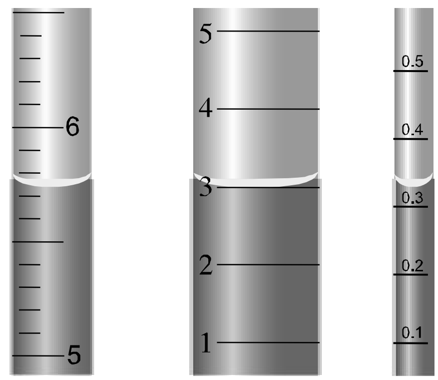
When using instruments with digital readouts you should record all the digits shown. The instrument has done the estimating for you.

When measuring liquids in narrow glass graduated cylinders, most liquids form a slight dip in the middle. This dip is called a *meniscus*. Your measurement should be read from the bottom of the meniscus. Plastic graduated cylinders do not usually have a meniscus. In this case you should read the cylinder from the top of the liquid surface. Practice reading the volume contained in the 3 cylinders below. Record your values in the space provided.

Left:________________

Middle: _____________

Right:_______________



SIGNIFICANT DIGITS

There are two kinds of numbers you will encounter in science, exact numbers and measured numbers. *Exact numbers* are known to be absolutely correct and are obtained by counting or by definition. Counting a stack of 12 pennies is an exact number. Defining 1 day as 24 hours are exact numbers. Exact numbers have an infinite number of significant digits.

Measured numbers, as we've seen above, involve some estimation. Significant digits are digits believed to be correct by the person making and recording a measurement. We assume that the person is competent in his or her use of the measuring device. To count the number of significant digits represented in a measurement we follow 2 basic rules:

1. If the digit is NOT a zero, it is significant.

2. If the digit IS a zero, it is significant if
   a. It is a sandwiched zero
      OR
   b. It terminates a number containing a decimal place

Examples:

3.57 mL has 3 significant digits (Rule 1)
288 mL has 3 significant digits (Rule 1)
20.8 mL has 3 significant digits (Rule 1 and 2a)
20.80 mL has 4 significant digits (Rules 1, 2a and 2b)
0.01 mL has only 1 significant digit (Rule 1)
0.010 mL has 2 significant digits (Rule 1 and 2b)
0.0100 mL has 3 significant digits (Rule 1 and 2b)
3.20×10^4 kg has 3 significant digits (Rule 1 and 2b)

SIGNIFICANT DIGITS IN CALCULATIONS

A calculated number can never contain more significant digits than the measurements used to calculate it.

Calculation rules fall into two categories:

1. <u>Addition and Subtraction</u>: answers must be rounded to match the measurement with the least number of decimal places.
   37.24 mL + 10.3 mL = 47.54 (calculator value), report as 47.5 mL

2. <u>Multiplication and Division</u>: answers must be rounded to match the measurement with the least number of significant digits.
   1.23 cm x 12.34 cm = 15.1782 (calculator value), report as 15.2 cm^2

DIMENSIONAL ANALYSIS

Throughout your study of science it is important that a unit accompanies all measurements. Keeping track of the units in problem can help you convert one measured quantity into its equivalent quantity of a different unit or set up a calculation without the need for a formula.

In conversion problems, equality statements such as 1 ft. = 12 inches, are made into fractions and then strung together in such a way that all units except the desired one are canceled out of the problem. Remember that defined numbers, such as the 1 and 12 above, are exact numbers and thus will not affect the number of significant digits in your answer. This method is also known as the Factor-Label method or the Unit-Label method.

To set up a conversion problem follow these steps.

1. Think about and write down all the "=" statements you know that will help you get from your current unit to the new unit.

2. Make fractions out of your "=" statements (there could be 2 fractions for each "="). They will be reciprocals of each other.

3. Begin solving the problem by writing the given amount with units on the left side of your paper and then choose the fractions that will let a numerator unit be canceled with a denominator unit and vice versa.

4. Using your calculator, read from left to right and enter the numerator and denominator numbers in order. Precede each numerator number with a multiplication sign and each denominator number with a division sign. Alternatively, you could enter all of the numerators, separated by multiplication signs, and then all of the denominators, each separated by a division sign.

5. Round your calculator's answer to the same number of significant digits that your original number had.

Example:
How many inches are in 1.25 miles?

Solution:

$$1\,\text{ft} = 12\,\text{in} \qquad \frac{1\,\text{ft}}{12\,\text{in}} \text{ OR } \frac{12\,\text{in}}{1\,\text{ft}}$$

$$5280\,\text{ft.} = 1\,\text{mile} \qquad \frac{5280\,\text{ft.}}{1\,\text{mile}} \text{ OR } \frac{1\,\text{mile}}{5280\,\text{ft.}}$$

$$1.25\,\cancel{\text{miles}} \times \frac{5280\,\cancel{\text{ft.}}}{1\,\cancel{\text{mile}}} \times \frac{12\,\text{in.}}{1\,\cancel{\text{ft.}}} = 79{,}200\,\text{in.}$$

As problems get more complex the measurements may contain fractional units or exponential units. To handle these problems treat each unit independently. Structure your conversion factors to ensure that all the given units cancel out with a numerator or denominator as appropriate and that your answer ends with the appropriate unit. Sometimes information given in the problem is an equality that will be used as a conversion factor.

Example: Suppose your automobile tank holds 23 gal and the price of gasoline is 33.5¢ per L. How many dollars will it cost you to fill your tank?

Solution: From a reference table we will find,
1 L = 1.06 qt
4 qt = 1 gal

We should recognize from the problem that the price is also an equality, 33.5¢ = 1 L
and we should know that 100¢ = 1 dollar

Setting up the factors we find,

$$23\,\cancel{\text{gal}} \times \frac{4\,\cancel{\text{qt}}}{1\,\cancel{\text{gal}}} \times \frac{1\,\cancel{\text{L}}}{1.06\,\cancel{\text{qt}}} \times \frac{33.5\cancel{¢}}{1\,\cancel{\text{L}}} \times \frac{\$1}{100\cancel{¢}} = \$29$$

In your calculator you should enter $23 \times 4 \div 1.06 \times 33.5 \div 100$ and get 29.0754717. However, since the given value of 23 gal has only 2 significant digits, your answer must be rounded to $29.

Squared and cubed units are potentially tricky. Remember that a cm^2 is really cm x cm. So, if we need to convert cm^2 to mm^2 we need to use the conversion factor 1 cm = 10 mm twice so that both centimeter units cancel out.

Example: One liter is exactly 1000 cm^3. How many cubic inches are there in 1.0 L?

Solution:
We should know that

$$1000\ cm^3 = 1\ L$$

From a reference table we find,

$$1\ in. = 2.54\ cm$$

Setting up the factors we find,

$$1.0\,\cancel{L} \times \frac{1000\,\cancel{cm} \times \cancel{cm} \times \cancel{cm}}{1\,\cancel{L}} \times \frac{1\,in}{2.54\,\cancel{cm}} \times \frac{1\,in}{2.54\,\cancel{cm}} \times \frac{1\,in}{2.54\,\cancel{cm}} = 61\,in^3$$

(The answer has 2 significant digits since our given 1.0 L contained two significant digits.)

As you become more comfortable with the concept of unit cancellation you will find that it is a very handy tool for solving problems. By knowing the units of your given measurements, and by focusing on the units of the desired answer you can derive a formula and correctly calculate an answer. This is especially useful when you've forgotten, or never knew, the formula!

Example: Even though you may not know the exact formula for solving this problem, you should be able to match the units up in such a way that only your desired unit does not cancel out.

What is the volume in liters of 1.5 moles of gas at 293 K and 1.10 atm of pressure?

The ideal gas constant is $\frac{0.0821\ L \bullet atm}{mol \bullet K}$

Solution: It is not necessary to know the formula for the ideal gas law to solve this problem correctly. Working from the constant, since it sets the units, we need to cancel out every unit except L. Doing this shows us that moles and kelvins need to be in the numerator and atmospheres in the denominator.

$$\frac{0.0821\ L \bullet \cancel{atm}}{\cancel{mol} \bullet \cancel{K}} \times \frac{1.5\,\cancel{mol}}{} \times \frac{293\,\cancel{K}}{} \times \frac{}{1.10\,\cancel{atm}} = 33\,L$$

(2 significant digits since our least accurate measuement has only 2 sig.digs.)

**NOTE: NEVER consider the number of significant digits in a constant to determine the number of significant digits for reporting your calculated answer. Consider ONLY the number of significant digits in given or measured quantities.

PURPOSE

In this activity you will review some important aspects of numbers in science and then apply those number handling skills to your own measurements and calculations.

MATERIALS

small cube
metric ruler
200 mL beaker
large graduated cylinder
spherical object
tweezers
flexible tape measure
balance

PROCEDURE

*Remember when taking measurements it is your responsibility to estimate a digit between the two smallest marks on the instrument.

1. Mass the small cube on a balance and record your measurement in the data table on your student page.

2. Measure dimensions (the length, width and height) of the small cube in centimeters, being careful to use the full measuring capacity of your ruler. Record the lengths in your data table.

3. Fill the 200 mL beaker with water to the 100 mL line. Carefully place the cube in the beaker and use the tweezers to gently submerge the cube. The cube should be just barely covered with water. Record the new, final volume of water.

4. Fill the large graduated cylinder ¾ of the way full with water. Record this initial water volume. Again, use the tweezers to gently submerge the cube and record the final water volume.

5. Mass the spherical object on a balance and record your measurement in the data table.

6. Use the flexible tape measure to measure the widest circumference of the sphere in centimeters. Be careful to use the full measuring capacity of the tape measure.

7. Fill the 200 mL beaker with water to the 100 mL line. Carefully place the spherical object in the beaker and, if needed, use the tweezers to gently submerge the sphere. Record the final volume of water from the beaker.

8. Fill the large graduated cylinder ¾ of the way full with water. Record this initial water volume. If needed, use the tweezers to gently submerge the sphere and record the new water volume.

9. Dry the cube and sphere and clean up your lab area as instructed by your teacher.

Name ______________________________

Period______________________________

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DATA AND OBSERVATIONS

<table>
<tr><th colspan="4">Data Table</th></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4" align="center">CUBE DATA</td></tr>
<tr><td colspan="4">Mass:</td></tr>
<tr><td>Dimensions</td><td>length:</td><td>width:</td><td>height:</td></tr>
<tr><td>Volume</td><td>Beaker
initial volume: 100 mL</td><td colspan="2">Beaker
final volume:</td></tr>
<tr><td></td><td>Graduated cylinder
initial volume:</td><td colspan="2">Graduated cylinder
final volume:</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4" align="center">SPHERE DATA</td></tr>
<tr><td colspan="4">Mass:</td></tr>
<tr><td>Dimensions</td><td colspan="3">Circumference:</td></tr>
<tr><td>Volume</td><td>Beaker initial volume: 100 mL</td><td colspan="2">Beaker final volume:</td></tr>
<tr><td></td><td>Graduated cylinder
initial volume:</td><td colspan="2">Graduated cylinder
final volume:</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">Formula for calculating the volume of a cube:</td></tr>
<tr><td colspan="4">Formula for calculating the circumference of a circle:</td></tr>
<tr><td colspan="4">Formula for calculating the diameter of a circle:</td></tr>
<tr><td colspan="4">Formula for calculating the volume of a sphere:</td></tr>
</table>

ANALYSIS

- Remember to follow the rules for reporting all data and calculated answers with the correct number of significant digits.
- You may need tables of metric and English conversion factors to work some of these problems.

1. For each of the measurements you recorded above, go back and indicate the number of significant digits in parentheses after the measurement. Ex: 15.7 cm (3sd)

2. Use dimensional analysis to convert the mass of the cube to
   a. mg

   b. ounces

3. Calculate the volume of the cube in cm^3.

4. Use dimensional analysis to convert the volume of the cube from cm^3 to m^3.

5. Calculate the volume of the cube in mL as measured in the beaker. Convert the volume to cm^3 knowing that $1\ cm^3 = 1\ mL$.

6. Calculate the volume of the cube in mL as measured in the graduated cylinder. Convert to cm^3 knowing that $1\ cm^3 = 1\ mL$.

7. Using the density formula $D = \frac{mass}{volume}$, calculate the density of the cube as determined by the
   a. ruler
   b. beaker
   c. graduated cylinder

8. Use dimensional analysis to convert these three densities into kg/m^3.

9. Convert the mass of the sphere to
   a. kg
   b. lbs.

10. Using the measured circumference, calculate the diameter of the sphere.

11. Calculate the radius of the sphere.

12. Calculate the volume of the sphere from its radius.

13. Calculate the volume of the sphere in mL as measured in the beaker. Convert to cm^3 knowing that 1 cm^3 = 1 mL.

14. Calculate the volume of the sphere in mL as measured in the graduated cylinder. Convert to cm^3 knowing that 1 cm^3 = 1 mL.

15. Using the density formula $D = \dfrac{mass}{volume}$, calculate the density of the sphere as determined by the
    a. tape measure

    b. beaker

    c. graduated cylinder

16. Use dimensional analysis to convert these three densities into lbs/ft^3.

CONCLUSION QUESTIONS

1. Compare the densities of the cube when the volume is measured by a ruler, beaker and graduated cylinder. Which of the instruments gave the most accurate density value? Use the concept of significant digits to explain your answer.

2. A student first measures the volume of the cube by water displacement using the graduated cylinder. Next, the student measures the mass of the cube before drying it. How will this error affect the calculated density of the cube? Your answer should state clearly whether the calculated density will increase, decrease or remain the same and must be justified.

3. A student measures the circumference of a sphere at a point slightly higher than the middle of the sphere. How will this error affect the calculated density of the cube? Your answer should state clearly whether the calculated density will increase, decrease, or remain the same and must be justified.